

MEASURABLE FOLIATIONS ASSOCIATED TO THE COADJOINT REPRESENTATION OF A CLASS OF SEVEN-DIMENSIONAL SOLVABLE LIE GROUPS

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We consider connected and simply connected seven-dimensional Lie groups whose Lie algebras have nilradical $\mathfrak{g}_{5,2}$ of Dixmier. First, we give geometric descriptions of the maximal-dimensional orbits in the coadjoint representation of all considered Lie groups. Next, we prove that, for each considered group, the family of the generic coadjoint orbits forms a measurable foliation in the sense of Connes. Finally, the topological classification of all these foliations is also provided.

MSC: 17B08, 17B30, 22E27, 53C12, 57R30

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1. Introduction

According to Dixmier [2, Proposition 1], the class of five-dimensional real or complex nilpotent Lie algebras consists of nine algebras which are denoted by $(\mathfrak{g}_1)^5$, $(\mathfrak{g}_1)^2 \oplus \mathfrak{g}_3$, $\mathfrak{g}_1 \oplus \mathfrak{g}_4$, $\mathfrak{g}_{5,1}$, $\mathfrak{g}_{5,2}$, $\mathfrak{g}_{5,3}$, $\mathfrak{g}_{5,4}$, $\mathfrak{g}_{5,5}$, $\mathfrak{g}_{5,6}$. Recently, we have classified seven-dimensional solvable Lie algebras with nilradicals $(\mathfrak{g}_1)^2 \oplus \mathfrak{g}_3$, $\mathfrak{g}_1 \oplus \mathfrak{g}_4$, $\mathfrak{g}_{5,2}$ and $\mathfrak{g}_{5,4}$ in [18]. By combining with [7, 9–12], the class of seven-dimensional solvable Lie algebras having five-dimensional nilradicals was classified. In association with [3, 4, 8], it creates a classification of seven-dimensional solvable Lie algebras. Note that the classification problem is a fundamental problem in Lie theory.