



# INTEGRABILITY IN A NONLINEAR MODEL OF SWING OSCILLATORY MOTION

SVETOSLAV G. NIKOLOV<sup>1,2</sup> and VASSIL M. VASSILEV<sup>1</sup>

Presented by Vassil M. Vassilev

Nonlinear dynamical systems can be studied in many different directions: i) finding integrable cases and their analytical solutions, ii) investigating the algebraic nature of the integrability, iii) topological analysis of integrable systems, and so on. The aim of the present paper is to find integrable cases of a dynamical system describing the rider and the swing pumped (from the seated position) as a compound pendulum. As a result of our analytical calculations, we can conclude that this system has two integrable cases when: 1) the dumbbell lengths and point-masses meet a special condition, 2) the gravitational force is neglected.

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