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BOOK REVIEW

Nonlinear Waves and Solitons on Contours and Closed Surfaces, Andrei Ludu, Springer, Cham, Third Edition 2022, xxxii+566 pages, ISBN 978-3-031-14640-4, doi: 10.1007/978-3-031-14641-1

Nowadays, the majority of mathematical models put forward to describe complex processes and phenomena observed in Nature adequately are basically nonlinear. Since the first observation of the *Wave of Translation* by *John Scott Russel* almost two centuries ago [5], and the long story until the introducing of the term *soliton* by *Zabusky and Kruskal* in 1965 [6], the theory of *Nonlinear Waves and Solitons* has achieved incredible progress an development although the nonlinearity is a great challenge for the researchers.

The essential feature distinguishing the current topical book by *Andrei Ludu* from the vast amount of other books published in this field in the last four decades (see, e.g., the classical books [1–4]) is the emphasis on the nonlinear solitary waves and solitons, in particular, evolving in compact spaces such as closed curves and contours as well as closed surfaces and interfaces. It should be stressed that there are still no other books presenting this theory consistently in which the results reported in the plurality of publications on nonlinear evolution equations on compact domains are summarized.

In the present book, the interested reader can find a sufficiently readable exposition of the basic ideas of this topic, taste the implementation of a wide range of analytical and numerical methods for obtaining new results in this field and enjoy a comprehensive review of their applications.

Complex systems are at the centre of attention of the book as they exhibit nonlinear behaviour leading to the existence of a wide diversity of localized patterns and self-organizing structures such as pulses having particle-like properties, loops, vertices, filaments, instantons, cyclic reactions, bifurcations, phase transitions, spontaneous symmetry breaking, etc.

The author reveals and makes use of the close relationship between the integrable systems of nonlinear evolution equations describing the dynamical behaviour of

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complex physical systems and the differential geometry of the related compact manifolds.

The focus is mainly on the mathematical aspects of the problems, but the book also presents and analysis a wide variety of nonlinear models, formulated mostly in the language of differential geometry, of processes and phenomena regarded and studied in classical mechanics and thermodynamics, quantum mechanics and field theory, models from particle physics to astrophysics ranging from microscopic to macroscopic scale. A number of the latest results on nonlinear solitary waves and solitons developing in closed, compact curves and surfaces are included in this third edition of the book. Many additional examples are given of systems and models in which the interaction between the nonlinearities and the boundaries is essential for the existence and dynamics of solitons.

As a whole, the book provides an introduction to the physics of solitons on compact structures for physicists, mathematicians and engineers. It is divided into four parts and consists of 19 chapters, each one including its own list of references.

Chapter 1 introduces the reader to the topic. Then, the first part of the book begins. It contains the fundamentals of topology and algebra, mathematical analysis and calculus on differentiable manifolds at a post-graduate level. Chapter 2 recalls some basic elements of topology with emphasis on the concept of compactness. Chapter 3 helps the reader to remind the basic points of the theory of differentiable manifolds, the underlying notions and fundamental theorems for the vector fields, differential forms, and derivatives of various types. Chapter 4 discusses the importance of boundaries.

The second part of the book deals with differential geometry of curves (Chapter 5) and surfaces (Chapter 6) in three dimensional Euclidean space and in more general manifolds. Special attention is paid to the theory of closed curves and curves lying on surfaces and to the action of differential operators on surfaces.

Chapters 7 and 8 are devoted to the theory of motion of curves and surfaces with an emphasis on the relationships between this theory and the theory of solitons and nonlinear integrable dynamical systems.

In the third part of the book, the focus is put on the application of the theory of solitons and nonlinear waves in fluid dynamics with the main interest directed to the closed curves (particle trajectories, stream lines) and surfaces (interfaces).

In the first place, in Chapters 9 and 10, the reader will find a concise exposition of the basic concepts, principles and governing equations (Euler and Navier–Stokes) of the kinematics and dynamics of fluids regarded as a continuous and homogeneous medium both in Lagrangian and Eulerian description. The presentation has a geometrical flavor arising from the formulations in terms of fibre bundles and the continuous usage of the language of differential geometry.

Chapter 11 presents some of the most famous nonlinear evolution equations in one space dimension together with basic elements of their derivation. These are the Korteweg-de Vries (KdV) equation, modified Korteweg-de Vries (mKdV) equation and its generalizations as well as the Boussinesq equation on a circle. Some of the well-known traveling wave solutions of these equations are also discussed and analyzed.

Elements of the theory of two-dimensional ideal flows are given in Chapter 12, the differential geometrical interpretation of this theory being discussed. Here, one can also find discussion on the solitary wave solutions of the Kadomtsev–Petviashvili equation in two space dimensions, description of two-dimensional fluid systems with moving boundaries and results about the oscillations and nonlinear waves in two-dimensional liquid drops.

The purpose of Chapter 13 is to introduce the reader to complex variables approach to the dynamics of free surfaces from the mechanics of droplets sight.

Chapters 14 and 15 offer other applications of soliton theory on compact surfaces in three dimensions like shapes of layered liquid drops or compact supported solitons, and consider the relationship between solitons and collective motions of nonlinear dynamical systems with boundaries and shape oscillations of liquid drops.

The forth part of the book provides various applications of the soliton theory on closed or bounded domains to systems at different physical scales, from elementary particles to plasma, magneto-hydrodynamics systems and neutron stars.

Chapter 16 is devoted to one of the most successful applications of the theory of nonlinear integrable systems related to the existence of solitons in the dynamics of shapes of one-dimensional nonlinear systems like filaments and polymer chains. Here, the relationship between the Riccati version of the Frenet–Serret equations for the vortex filaments and the cubic nonlinear Schrödinger equation is also traced and commented.

Chapter 17 introduces various applications of solitons and instantons on the boundaries of microscopic systems, in the theory of elementary particles and quantum fields, in the description of exotic shapes of heavy nuclei, in the investigation of the phenomenon of exotic radioactivity and relationships between solitons on closed curves and quantum Hall drops.

Chapter 18 reports the results of application of the closed contour dynamics and some fundamental theorems from differential geometry to several types of macroscopic systems. First, the author presents studies of the geometry of trajectories of charged particles in magnetic fields and solutions of problems concerning the trapping of particles inside closed magnetic surfaces. Then, he focuses on the occurrence of localized stable waves orbiting around elastic spheres. The chapter ends with a discussion on the nonlinear evolution of oscillation modes in neutron stars.

A short mathematical Appendix is given in the last Chapter 19.

In conclusion, I would say that the third edition of the book *Nonlinear Waves and Solitons on Contours and Closed Surfaces* by *Andrei Ludu* is worth reading for everyone interested in the exciting theory of solitary waves and its myriad of applications.

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