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WEYL GEOMETRIC APPROACH TO THE GRADIENT-FLOW EQUATIONS IN INFORMATION GEOMETRY

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The gradient-flow equations with respect to the potential functions in information geometry are reconsidered from the perspective of the Weyl integrable geometry. The pre-geodesic equations associated with the gradient-flow equations are regarded as the general pre-geodesic equations in the Weyl integrable geometry.

MSC: 53E99, 53Z05
Keywords: Dually-flat structure, gradient-flow equations, information geometry, Weyl integrable geometry

1. Introduction

Information geometry (IG) [1] is a useful method exploring the fields of information science by means of differential geometry. IG is invented from the studies on invariant properties of a manifold of probability distributions, and a modern treatment is based on the affine differential geometry [12]. IG has been applied to the different fields including statistical physics [5, 6], statistics, dynamical systems [8, 13] and so on. It is known that the gradient-flow equations are useful for some optimization problems. A recent attentions is the gradient-flows in metric spaces [2]. The gradient flows on a Riemann manifold follow the direction of gradient descent (or ascent) in the landscape of a potential functional, with respect to the curved structure of the underlying metric space. The information geometric studies on the gradient systems were originally studied by Nakamura [13], Fujiwara and Amari [8]. A remarkable feature of their works is that a certain kind of gradient flow on a dually flat space can be expressed as a Hamilton flow. Specifically, the linear differential equations

$$\frac{\mathrm{d}\eta_j^{\mathrm{gf}}}{\mathrm{d}t} = -\eta_j^{\mathrm{gf}}, \qquad j = 1, 2, \ldots, 2m \tag{1}$$

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