ON THE DYNAMICS OF THE SOLAR SYSTEM IV: PERIHELION PRECESSION AND ECCENTRICITY EVOLUTION

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The perihelion precession rate and the time derivative of the orbital eccentricity are joined into the derivative of a complex variable being representative of the Runge-Lenz vector. The integration of the linear differential equation system so obtained yields the evolution of the perihelion and the eccentricity of all the planets. Each eccentricity reaches a maximum, and in the case of giant planets also a minimum. The variation of each semimajor axis is shown to be very small. Since the semimajor axes are bounded and the planetary orbits never intersect, the stability of the solar system is proven.

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