



SPECTRAL GEOMETRY OF UNDULOIDS

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Communicated by John Oprea

This paper examines the eigenvalues and eigenfunctions of the Laplace operator associated with a set of mathematically defined surfaces which can be produced experimentally by attaching two equally sized rings to opposite poles of a soap bubble and separating the rings. The shapes produced are called unduloids. These calculations show 1) for a range of ring sizes, as a function of ring separation, the first indexed eigenvalue has a minimum, pointing to an “optimum” shape, and 2) given the eigenfunctions in the form of their differential equations and a preference for symmetry, the underlying unduloid geometry may be deduced.

MSC: 35R30, 53A05, 53A10, 53C42, 58J50

Keywords: Inverse problem, minimal surfaces, soap bubbles, spectral geometry, unduloid

Contents

1	Introduction	48
2	Surface Geometry	50
3	Derivation of Arc Length Parameterized Eigenfunctions	52
3.1	Partial Differential Equation	52
3.2	Separation of Variables	52
4	How Eigenfunctions Reveal the Unduloid Geometry	53
4.1	First Change of Variables: Landen’s Transformation	53
4.2	Second Change of Variables: Elliptic Integral Recalls Nonlinear Pendulum Giving a Weighted Sum to an Angular Variable	56
4.3	Relationship between the Variables u and $\psi = s/a$	56
4.4	Prioritizing Symmetry	57
4.5	Parameterization Matches that from Cartan Moving Frame	58
4.6	Metric Tensor Components from the Elliptic Parameterization	59
5	Numerical Method to Solve for the Eigenvalues and Eigenfunctions	60
5.1	Eigenvalues	60
5.2	Eigenfunctions	62
6	Conclusions	63
	References	64