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ON FOCK TRANSFORM AND MOSER REGULARIZATION MAP

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For the Kepler problem, the Fock transform is the starting point of the quantization in the momentum representation, and its associated Moser regularization map is an imbedding of energy surfaces that yields the dynamic completion. Here, we first introduce a version of the Fock transform, with the normalization chosen so that the analytic formula for the resulting Moser regularization map exhibits a uniform dependence on the energy. Next, we provide a characterization for the Moser regularization map in terms of contact form and symmetries.

MSC: 53D10, 70H06
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Contents

1. Introduction

For an integer $n \geq 2$, we denote the Euclidean space $\mathbb{R}^n$ without the origin as $\mathbb{R}^n_*$. Additionally, we have the identification $\Sigma := T^*\mathbb{R}^n_* \equiv T\mathbb{R}^n_*$ via the standard Euclidean metric on $\mathbb{R}^n$. Let p represent the magnitude of the momentum vector $\vec{p}$, and q denote the magnitude of the position vector $\vec{q}$.

The Hamiltonian H for the Kepler problem in dimension n is a smooth real function defined on the phase space Σ, mapping $(\vec{q}, \vec{p})$ to $\frac{1}{2}p^2 - \dfrac{1}{q}$, i.e., $H = \frac{1}{2}p^2 - \dfrac{1}{q}$.

In the Kepler problem, there exist two types of trajectories: conical and colliding. Unlike conical trajectories, colliding trajectories "blow up" in a finite amount

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