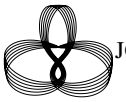
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EXPLICIT SOLUTIONS OF TWO-CENTER PROBLEM IN THE PLANE: NEGATIVE ENERGIES

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Presented by Ivaïlo M. Mladenov

In 1760 Euler proved the integrability of the planar problem of two fixed centers of gravitation. Later Jacobi separated the variables and integrated the equations of motion in terms of elliptic coordinates. In 1860 Köningsberger found solutions expressed by the four Jacobi's theta functions. These solutions contain complete Abelian integrals and are not convenient for use.
We present effective solutions in terms of the classical Jacobi's elliptic functions sn, cn and dn. Their arguments depend linearly on some pseudo-time and on certain elliptic modulus. It is proved that, if the energy of the system is negative, there exist exactly ten topologically different types of regular solutions. We illustrate graphically and comment each of these ten cases.

MSC: 34A05, 70H06, 70H12
Keywords: Explicit solutions, negative energies, two centers of gravitation

Contents

1. Introduction

One of the famous integrable problems of the classical mechanics is the problem of two centers of gravitation, i.e., the problem of determining the motion of a