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EXPLICIT PARAMETERIZATIONS AND DECOMPOSITIONS OF SYMPLECTIC MATRICES IN \mathbb{R}^4

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We present a plethora of explicitly parameterized real symplectic matrices in dimension four which were missing up to now. Structurally, these matrices depend on a set of ten real parameters and split naturally as an union of squeezing, rotational and boost transformations, and their fundamental representations have been derived relying on exponential, Cayley and Fedorov-like maps.

Explicit formulas relating the Lie algebra and the Lie group elements in both directions were found. Besides, an algorithmic procedure for factorization of an arbitrary symplectic matrix as a product of three matrices with clear mathematical/physical interpretations was found and exemplified via numerous matrices. Surprisingly, as a side effect of the decomposition, we have arrived at alternative realizations of the real ortho-symplectic matrices in four dimensions.

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