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A CLASS OF REPRESENTATIONS OF THE $\mathbb{Z}_2 \times \mathbb{Z}_2$-GRADED SPECIAL LINEAR LIE SUPERALGEBRA $\mathfrak{sl}(m_1+1, m_2|n_1, n_2)$ AND QUANTUM STATISTICS

NELI I. STOILOVA and JORIS VAN DER JEUGT

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The description of the $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded special linear Lie superalgebra is carried out via a set of generators that satisfy triple relations and are called creation and annihilation operators. With respect to these generators, a class of Fock type representations of the algebra is constructed. The properties of the underlying statistics are discussed and its Pauli principle is formulated.

MSC: 17B70, 17B75, 81R05
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1. Introduction

A result from quantum field theory is that particles with half-integer spins are fermions, satisfying the Fermi-Dirac statistics, and particles with integer spins are bosons, satisfying the Bose-Einstein statistics. However, beyond Fermi-Dirac and Bose-Einstein statistics, various kinds of generalized quantum statistics have been introduced, investigated and discussed (see, e.g., [12] for a review and references). One of the first such generalizations are the so called parafermion and paraboson statistics [2]. The algebraic structure behind a system of $2m$-parafermion operators is the orthogonal Lie algebra $\mathfrak{so}(2m+1)$ [5, 11], and behind a system of $2n$-parabosons is the orthosymplectic Lie superalgebra $\mathfrak{osp}(1|2n)$ [1]. For a mixed system of $2m$-parafermions and $2n$-parabosons there are two types of mutual non-trivial relations (from a physical point of view) [3] with algebraic structures the Lie superalgebra $\mathfrak{osp}(2m+1|2n)$ [8] and the $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded Lie superalgebra $\mathfrak{osp}(1, 2m|2n, 0) \equiv \mathfrak{pso}(2m+1|2n)$ [14, 17]. All these algebras are of type B Lie algebras, Lie superalgebras or $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded Lie superalgebras. Furthermore, generalized statistics have been associated with all classical Lie algebras and basic classical Lie superalgebras from the infinite series A, B, C, and D and are referred to as A, B, C and D-(super)statistics [7, 15, 16]. Therefore it is natural to consider their $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded counterparts.

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