



SOLITON HIERARCHIES ASSOCIATED WITH LIE ALGEBRA $\mathfrak{sp}(6)$

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In this paper, by selecting appropriate spectral matrices within the loop algebra of symplectic Lie algebra $\mathfrak{sp}(6)$, we construct two distinct classes of integrable soliton hierarchies. Then, by employing the Tu scheme and trace identity, we derive the Hamiltonian structures of the aforementioned two classes of integrable systems. From these two classes of integrable soliton hierarchies, we select one particular hierarchy and employ the Kronecker product to construct an integrable coupling system.

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1. Introduction

Soliton theory and integrable systems represent a significant branch of applied mathematics and mathematical physics, characterized by a rich variety of content and research methodologies. On the one hand, soliton equations may be derived through geometric approaches or by employing geometric tools; on the other hand, the algebraic properties of integrable systems can be effectively studied using Lie algebra theory. In recent years, rapid developments in mathematical physics and