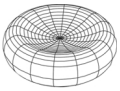
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RIGOROUS RECONSTRUCTION OF A 3D ROTATIONAL MATRIX FROM ANY FIVE OF ITS ELEMENTS

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In this paper we present a straightforward algorithm that reconstructs the entire three-dimensional rotational matrix from the knowledge of any five of its elements. Additionally, we have demonstrated the application of the proposed scheme via various concrete examples of incomplete matrices.

MSC: 15A90, 17B81, 22E70, 81R05
Keywords: Five-dimensional Hopf parameters, kinematic differential equations, quaternions, Rodrigues' vector, stereographic projection, Tait-Bryan angles

Contents

1. Introduction

Rotational matrices in three-dimensional Euclidean space have been of interest since 1776, when Euler showed that these matrices are in fact a three-dimensional manifold and therefore we need to know only three independent elements of the rotational matrix in order to determine it unambiguously. This can be seen as follows. Any 3×3 rotational matrix, which belongs to the special orthogonal group SO(3,$\mathbb{R}$), has nine elements. However, these elements are not independent. According to their very definition

$$\mathcal{R}^t\mathcal{R} = \mathcal{R}\mathcal{R}^t = I_3, \qquad \mathcal{R} \in \mathrm{SO}(3,\mathbb{R}) \tag{1}$$

special orthogonal matrices presented above must satisfy the following set of geometrical constraints.