



ROTATIONAL MATRICES IN 3D AS SHADOWS OF ORTHO-SYMPLECTIC MATRICES IN 4D

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This paper starts with a short historical review of the appearance of the rotational matrices in science which principle purpose is to display them explicitly in their original form and notation. Then we continue with reviewing the symplectic matrices in four-dimensional real space with emphasis on a special class of them – the so-called ortho-symplectic matrices. Further on, we present a plethora of uniformly generated rotational matrices in 3D following the scheme described in a Flowchart.

This scheme is based on the Hopf map which presents the Euclidean space as a shadow of the four-dimensional real space. Viewed in this way, it is easy to be seen that the rotational motions in the total space is transmitted as rotational motions in the base space and this is used to built up any three-dimensional rotational matrix (including the symmetrical ones).

Additionally, a theorem is proved which says that all 3D rotational matrices can be obtained in this manner. On the way it is pointed out also that this opens the possibility to describe easily their composition and to generate new 3D rotational matrices.

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