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DIFFERENTIAL INVARIANTS OF SUBMERSIONS WITH RESPECT TO TRANSFORMATION GROUPS

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The main subject of this paper are the second-order differential invariants of submersions with respect to the group of conformal transformations of Euclidean spaces. In particular, it is proved that the ratio of principal surface curvatures is a second-order differential invariant with respect to the group of conformal transformations.

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1. Introduction

In his Erlangen program Felix Klein proposed a unified approach to the description of various geometries. According to this program, one of the main problems of geometry is the construction of invariants of geometric objects with respect to the action of the group defining this geometry. The construction of invariants is based on the ideas of Sophus Lie, who introduced the Lie groups into the geometry. In particular, when considering classification problems and equivalence problems in differential geometry, one should consider differential invariants with respect to the action of Lie groups. In this case, the problem of equivalence of geometric objects is reduced to finding the complete system of scalar differential invariants [5].

The definition of a differential invariant of order k as a function on the space of k-jets of sections of the corresponding bundle made it possible to work effectively with them, and using invariant differentiation, new invariants can be obtained from known differential invariants.

Depending on the geometry, the orders of the first non-trivial differential invariants can be different. For example, in the space $\mathbb{R}^3$ equipped with the Euclidean metric, the complete system of differential invariants of a curve is its curvature and torsion, which are second and third order invariants, respectively. The first differential

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