



CYCLIC NUMBER REPRESENTATION AND DFT STRUCTURE OF POLYNOMIAL EQUATIONS

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This paper presents a structural reformulation of the polynomial root-finding problem from the perspective of representation theory of finite abelian groups and the Discrete Fourier Transform (DFT). By embedding polynomials into cyclic number algebras, we demonstrate that the global configuration of polynomial roots can be understood as the decomposition of the regular representation of cyclic groups in the frequency domain, with DFT serving as the natural tool for this decomposition. This yields a structural method for analyzing root distributions and constructing numerical initial values, serving as a beneficial supplement to traditional numerical root-finding algorithms. The framework is extended to multivariate polynomial systems, demonstrating potential adaptability in parallel computing environments. Numerical experiments demonstrate positive effects on initial value selection and iteration stability.

MSC: 15A18, 43A25, 12F10

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