



SINGULAR RIEMANNIAN FOLIATIONS AND CUBIC DYNAMICS ON $\mathbb{RP}^{2m-1}$

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We study a multiparameter family of cubic vector fields on odd-dimensional spheres and their real projective quotients. The construction extends a previously considered two-parameter case to arbitrary dimension. The fields are odd, tangent to the sphere, and therefore induce smooth vector fields on real projective space. We prove that the squared coordinate radii are first integrals and that the resulting dynamics preserves a parameter-independent toric singular Riemannian foliation, while the parameters change only the angular frequencies on its leaves.

MSC: 14P05, 37C10, 53C12, 53C20, 57R30

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