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# GEOMETRY AND COMBINATORICS OF SIX–POINT TRILOGARITHM IDENTITY

MOHAMMAD R. RAHMATI and ZACARÍAS MALACARA-HERNÁNDEZ

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We give a geometric and cohomological proof of Goncharov's six–point trilogarithm identity using logarithmic differential forms on the Deligne–Mumford compactification of the moduli space of six marked points on the projective line. We construct a canonical logarithmic three-form on the open moduli space and show that the vanishing of its total Poincaré residue, via the global residue theorem, is equivalent to the universal six–point functional equation in weight three. Using Orlik–Solomon theory for hyperplane bi-arrangements, this relation is interpreted cohomologically and motivically through a boundary identity in the scissors-congruence coalgebra.

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## Contents



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