



GEOMETRY AND COMBINATORICS OF SIX-POINT TRILOGARITHM IDENTITY

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We give a geometric and cohomological proof of Goncharov’s six-point trilogarithm identity using logarithmic differential forms on the Deligne–Mumford compactification of the moduli space of six marked points on the projective line. We construct a canonical logarithmic three-form on the open moduli space and show that the vanishing of its total Poincaré residue, via the global residue theorem, is equivalent to the universal six-point functional equation in weight three. Using Orlik–Solomon theory for hyperplane bi-arrangements, this relation is interpreted cohomologically and motivically through a boundary identity in the scissors-congruence coalgebra.

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