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DEFORMATIONS OF MINIMAL SURFACES

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Abstract. Here we combine group-theoretical and differential-geometric techniques for considerations of minimal surface deformations in the ordinary three-dimensional space. This approach allows a consideration of a novel family of transformations generated by complex rotations. The resulting generalized deformations are compared with the well-known Bonnet and Goursat transformations and illustrated via Schwarz skew quadrilateral to provide a clarification of their geometrical origin.

1. Introduction

The most fundamental quantities of the theory of smooth surfaces in ordinary three-dimensional space are the Gaussian, respectively the mean curvature of the surface. The vanishing of the first quantity selects the class of the so-called flat surfaces while the vanishing of the later distinguishes the class of really remarkable surfaces known as “minimal”. The term “flat” is coined here because the prime example of a flat surface is the plane. The history of the minimal surfaces began more than two centuries ago with answering the following question (raised by Lagrange in connection with his studies of the variational problems): “What does the surface bounded by a given contour look like when it has the smallest surface area?”. This variational problem leads to a partial differential equation which turns out to be just the minimal surface equation (for more details cf. Darboux 1914; Osserman 1986; Karcher 1989; Jost 1994 and Oprea 1997).

In view of the fundamental observation in the minimal surface theory that every minimal surface belongs to one-parameter family of minimal surfaces, the so-called Bonnet family, the situation with the other quite interesting transformation discovered by Goursat is a little strange. While Bonnet transformation have