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GENERALIZED ACTIONS

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Abstract. In this paper a generalization of the concept of action is considered. This notion is based on a new algebraic structure called generalized groups. An action is deduced by imposing an Abelian condition on a generalized group. Generalized actions on normal generalized groups are also considered.

1. Basic Notions

The theory of generalized groups was first introduced in [1]. A generalized group means a non-empty set G admitting an operation

$$\begin{aligned} G \times G &\to G \\ (a, b) &\mapsto ab \end{aligned}$$

called multiplication which satisfies the following conditions:

i) $(ab)c = a(bc)$ for all a, b, c in G;
ii) For each $a \in G$ there exists a unique $e(a) \in G$ such that $ae(a) = e(a)a = a$;
iii) For each $a \in G$ there exists $a^{-1} \in G$ such that $aa^{-1} = a^{-1}a = e(a)$.

Theorem 1.1. [1] For each $a \in G$ there exists a unique $a^{-1} \in G$.

Theorem 1.2. [2] Let G be a generalized group and $ab = ba$ for all a, b in G. Then G is a group.

Example 1.1. Let $G = \mathbb{R} \times \mathbb{R} \backslash \{0\} \times \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Then G with the multiplication $(a_1, b_1, c_1)(a_2, b_2, c_2) = (b_1 a_1, b_1 b_2, b_1 c_2)$ is a generalized group.