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THE VARIATIONAL PRINCIPLE OF HERGLOZ AND RELATED RESULTS

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Abstract. This is a review of the variational principle proposed by Gustav Hergotz and resent results related to it. In that variational principle the functional is defined by a certain differential equation instead of an integral. The solutions of the equations for the extrema of the functional determine contact transformations. Some of those results are: two Noether-type theorems for finding conserved quantities and identities, a method for calculating symmetry groups of the functional and several applications.

1. Introduction

In the 1930-s Gustav Herglotz proposed a *generalized variational principle* with one independent variable, which generalizes the classical variational principle by defining the functional, whose extrema are sought, by a certain ordinary differential equation. Herglotz variational principle contains the classical variational principle as a special case. His original idea was published in 1979 in his collected works [8] and [9]. It is especially suitable for a variational description of nonconservative processes. It can give a variational description of such processes even when the Lagrangian is not dependent on time, something which can not be done with the classical variational principle. It is also closely related to contact transformations.

The generalized variational principle of Herglotz defines the functional z, whose extrema are sought, by the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} = L\Big(t, x(t), \frac{\mathrm{d}x(t)}{\mathrm{d}t}, z\Big) \tag{1}$$

where t is the independent variable, and $x(t) \equiv (x_1(t), \dots, x_n(t))$ stands for the argument functions. In order for the equation (1) to define a functional $z = z[x]$ of $x(t)$ it has to be solved with the same fixed initial condition $z(0)$ for all argument