STAR PRODUCTS AND APPLICATIONS*

MARI IIDA and AKIRA YOSHIOKA

Department of Mathematics, Tokyo University of Science, 162-8601 Tokyo, Japan

Abstract. A family of star products parametrized by complex matrices is defined. Especially commutative associative star products are treated, and star exponentials with respect to these star products are considered. Jacobi's theta functions are given as infinite sums of star exponentials. As application, several concrete identities are obtained by properties of the star exponentials.

1. Star Products

Using an arbitrary complex symmetric matrix, we can define a star product, which gives a family of star products parameterized by complex matrices [4–6]. In particular for symmetric matrices we obtain a family of commutative associative star products [1, 2].

In this note, as a special case we consider a family of star product algebras of functions of one variable. Using star exponentials of these algebras we describe Jacobi's theta and its basic identities (cf. [1, 2, 6]).

First we consider a star product given by an arbitrary complex matrix. For simplicity, we consider star products of two variables (u_1, u_2). The general case for $(u_1, u_2, \cdots, u_{2m})$ is similar.

For any 2×2 complex matrix $\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \in M_2(\mathbb{C})$, we have a biderivation in the space of polynomials

$$p_1 \overleftarrow{\partial} \Lambda \overrightarrow{\partial} p_2 = p_1 \left(\sum_{\alpha\beta} \lambda_{\alpha\beta} \overleftarrow{\partial_\alpha} \overrightarrow{\partial_\beta} \right) p_2 = \sum_{\alpha\beta} \lambda_{\alpha\beta} \partial_\alpha p_1 \, \partial_\beta p_2, \quad p_1, p_2 \in \mathcal{P}(\mathbb{C}^2).$$

Then we define a star product by the formula

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