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AN APPLICATION OF PROLONGATION ALGEBRAS TO DETERMINE BÄCKLUND TRANSFORMATIONS FOR NONLINEAR EQUATIONS

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Abstract. Prolongation algebras which are determined by applying a version of the Wahlquist-Estabrook method to three different nonlinear partial differential equations can be employed to obtain not only Lax pairs but Bäcklund transformations as well. By solving Maurer-Cartan equations for the related group specified by the prolongation algebra, a set of differential forms is obtained which can lead directly to these kinds of results. Although specific equations are studied, the approach should be applicable to large classes of partial differential equations.

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Keywords: algebra, Bäcklund transformation, differential form, nonlinear equation, prolongation

1. Introduction

Surfaces which have constant Gaussian curvature are of great interest for a variety of reasons. There is a correspondence between solutions of certain nonlinear partial differential equations and manifolds of constant Gaussian curvature. The types of equations which pertain to constant Gaussian curvature are in fact $\sinh$ and sine-Gordon type equations. These are nonlinear partial differential equations and possess solutions which will have a solitonic character in general [1, 9, 11]. If a particular solution for one of these equations can be obtained, it can then be used to obtain a surface in, for example, three space by means of the structure equations for a two-dimensional manifold [10]. Moreover, by formulating the equations