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# ON NEW IDEAS OF NONLINEARITY IN QUANTUM MECHANICS

VASYL KOVALCHUK

*Institute of Fundamental Technological Research, Polish Academy of Sciences*
*$5^B$, Pawińskiego Str., 02-106 Warsaw, Poland*

**Abstract.** Our main idea is to suggest a new model of non-perturbative and geometrically motivated nonlinearity in quantum mechanics. The Schrödinger equation and corresponding relativistic linear wave equations derivable from variational principles are analyzed as usual self-adjoint equations of mathematical physics. It turns out that introducing the second-order time derivatives to dynamical equations, even as small corrections, can help to obtain the regular Legendre transformation. Following the conceptual transition from the special to general theory of relativity, where the metric tensor loses its status of the absolute geometric object and becomes included into degrees of freedom (gravitational field), in our treatment the Hilbert-space scalar product becomes a dynamical quantity which satisfies together with the state vector the system of differential equations. The structure of obtained Lagrangian and equations of motion is very beautiful, as usually in high-symmetry problems.

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*Keywords*: non-perturbative nonlinearity, self-adjoint dynamical equations, Schrödinger equation, highly symmetric problems, Hilbert-space scalar product

## Introduction

It is well known that quantum mechanics is still plagued with some paradoxes concerning

- decoherence
- measurement process
- reduction of the state vector.