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SECOND ORDER SYMMETRIES OF THE CONFORMAL LAPLACIAN

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Abstract. Let (M, g) be an arbitrary pseudo-Riemannian manifold of dimension at least three. We determine the form of all the conformal symmetries of the conformal Laplacian on (M, g), which are given by differential operators of second order. They are constructed from conformal Killing two-tensors satisfying a natural and conformally invariant condition. As a consequence, we get also the classification of the second order symmetries of the conformal Laplacian. We illustrate our results on two families of examples in dimension three. Besides, we explain how the (conformal) symmetries can be used to characterize the R-separation of some PDEs.

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1. Introduction

We work over a pseudo-Riemannian manifold (M, g) of dimension $n \geq 3$, with Levi-Civita connection ∇ and scalar curvature Sc. Our main result is the classification of all the differential operators D_1 of second order such that the relation

$$\Delta_Y D_1 = D_2 \Delta_Y \tag{1}$$

holds for some differential operator D_2, where $\Delta_Y := \nabla_a \mathrm{g}^{ab} \nabla_b - \frac{n-2}{4(n-1)} \mathrm{Sc}$ is the conformal Laplacian. Such operators D_1 are called conformal symmetries of order two of Δ_Y. They preserve the kernel of Δ_Y, i.e., the solution space of the equation $\Delta_Y \psi = 0$, $\psi \in \mathcal{C}^\infty(M)$.