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A NEW CHARACTERIZATION OF EULERIAN ELASTICAS

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Abstract. Here we present a new characterization of Euler elastica via the Weierstrassian functions.

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1. Euler Elastica and the Problem of Pure Bending

The problem of pure bending of elastic rods and laminae in its general form was stated for the first time in a clear mathematical formulation by Daniel Bernoulli. In 1742, in a letter to Leonhard Euler, Daniel Bernoulli suggested to him to find all the possible equilibrium shapes assumed by an elastic lamina (or rod) under two bending terminal loads (Fig. 1). On the base of the ideas of his uncle – James Bernoulli, Daniel Bernoulli suggested also an expression for the total potential energy of the lamina – an integral of the squared curvature over the profile curve of the bent lamina (see the integral in (1)), and raised himself the conjecture of the minimum potential energy of the lamina in equilibrium.

Euler presented the solution two years later in the appendix to his historical treatise on the calculus of variations, published in 1744. Following Bernoulli's suggestions, Euler made use of the "isoperimetric method", as it was named at that days the calculus of variations technique. The equation that he obtained – the *Euler-Lagrange equation*, was a fourth order ordinary differential equation. Euler succeeded in integrating it to a first order equation and obtained an integral-form solution. Actually it was an elliptic integral, which Euler went on analyzing from