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ON THE GEOMETRY INDUCED BY LORENTZ TRANSFORMATIONS IN PSEUDO-EUCLIDEAN SPACES

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Abstract. The Lorentz transformations of order (m, n) in pseudo-Euclidean spaces with indefinite inner product of signature (m, n) are extended in this work from $m = 1$ and $n \geq 1$ to all $m, n \geq 1$. A parametric realization of the Lorentz transformation group of any order (m, n) is presented, giving rise to generalized gyrogroups and gyrovector spaces called bi-gyrogroups and bi-gyrovector spaces. The latter, in turn, form the setting for generalized analytic hyperbolic geometry that underlies generalized balls called eigenballs.

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1. Introduction

The Lorentz transformations $\Lambda \in \mathrm{SO}(1, \mathrm{n})$ of special relativity are transformations of time-space points $(t, \mathbf{x})$, $t \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$, of a pseudo-Euclidean space $\mathbb{R}^{1,n}$ with inner product of signature $(1, n)$. In physical applications $n = 3$, but in applications in geometry we allow $n \in \mathbb{N}$. The Lorentz transformation group $\mathrm{SO}(1, \mathrm{n})$ is a group of special linear transformations in $\mathbb{R}^{1,n}$ that leave the inner product invariant. They are special in the sense that the determinant of the $(1 + n) \times (1 + n)$ matrix representation of each $\Lambda \in \mathrm{SO}(1, \mathrm{n})$ is 1 and its $(1, 1)$ entry is positive.

A parametric realization of the Lorentz transformation group $\mathrm{SO}(1, \mathrm{n})$ in terms of the two parameters $V \in \mathbb{R}^n_c = \{V \in \mathbb{R}^n ; \|V\| < c\}$ and $O_n \in \mathrm{SO(n)}$ is presented in [9], where $\mathbb{R}^n_c$ is the ball of all relativistically admissible velocities