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AXIALLY SYMMETRIC WILLMORE SURFACES DETERMINED BY QUADRATURES

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Abstract. The work is concerned with a special family of axially symmetric surfaces providing local extrema to the so-called Willmore functional, which assigns to each surface its total squared mean curvature. The components of the position vector of the profile curves of the regarded Willmore surfaces satisfy a system of first-order ordinary differential equations. The solutions of this system are expressed by quadratures in terms of the tangent angle and, in this way, the corresponding Willmore surfaces are determined.

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1. Introduction

The functional

$$\mathcal{W} = \int_{\mathcal{S}} H^2 \mathrm{d}A \tag{1}$$

which assigns to each surface $\mathcal{S}$ immersed in the three-dimensional Euclidean space its total squared mean curvature H, $\mathrm{d}A$ being the induced surface element, was proposed about two centuries ago by the prominent French scientists Siméon Denis Poisson and Marie-Sophie Germain as the bending energy of thin elastic shells [6, 9]. Nowadays, however, it is widely known as the Willmore functional (energy) due to the work [16] published in 1965 by the English geometer Thomas James Willmore (see also [17]).