Eighteenth International Conference on
Geometry, Integrability and Quantization
June 03–08, 2016, Varna, Bulgaria
Ivaïlo M. Mladenov, Guowu Meng
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2017, pp 203–216
doi: 10.7546/giq-18-2017-203-216

Geometry, Integrability and Quantization XVIII

TRAJECTORIES OF THE PLATE-BALL PROBLEM

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Abstract. The plate-ball problem concerns the shortest trajectories traced by a rolling sphere on a horizontal plane between the prescribed initial and final states meaning the positions and orientations of the sphere. Here we present an explicit parametric representation of these trajectories in terms of the Jacobian elliptic functions and elliptic integrals.

MSC: 70E18, 70Q05, 81E15
Keywords: curvatures, trajectories, Euler elasticas, plate-ball problem

1. Statement of the Problem

The problem of finding the shortest paths (optimal curves) traced by a spherical ball while rolling without slipping about a horizontal axis on an infinite horizontal plate was stated by Hammersley [8] in 1983. Since then this problem, as well as its variants and generalizations, have become widely known as the *plate-ball problem*. In the version we are going to consider we have looked for a curved path with a minimum length that delivers a sphere on a horizontal plane between two prescribed initial and final states meaning the positions and orientations of the sphere. Following Hammersley's original formulation in quaternions and calculus of variations settings, Arthurs and Walsh [2] showed, by making use of the Pontryagin maximum principle, that the problem has a solution which is readily expressed via elliptic functions. Their approach leads to an intrinsic equation for the curvature of the shortest paths. Here we give an explicit solution of this equation in terms of the Jacobian elliptic functions and elliptic integrals thereby parameterizing the optimal curves of the considered plate-ball problem.