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# ON GEODESIC BIFURCATIONS

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**Abstract.** In this paper we study fundamental equations of geodesics on surfaces of revolution. We obtain examples of existence of geodesic bifurcation.

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*Keywords*: Geodesic bifurcation, surfaces of revolution

## 1. Introduction

The theory of geodesics began by Bernoulli and Euler. Due to this problem the variation calculus has started. Geodesics are extremals of lenght on surfaces and also on Riemannian and pseudo-Riemannian manifolds. Detailed description of this problem can be found in [1–6, 9, 13–19].

Lagrange obtained new properties of geodesics on surfaces, e.g. he proved that ideal rubber (spring) on surface has a position of the geodesic, [8].

It is well-know that there exist one and only one geodesic going throught given point in given direction. This statement is valid for surfaces where Christoffel symbols are diferentiable. The proof follows from analysis of ordinary differential equations.

On the other hand, if the Christoffel symbols are continuous, then geodesics exist for above mentioned. We demonstrate an example of connections which components are not differentiable, but geodesics have common properties, that is there do not exist bifurcations.

Further, in this paper we obtained example of surface of revolution where exist more than one geodesic going through given point in given direction, i.e., a geodesic bifurcation exists.