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DIAGONALIZATION OF HAMILTONIANS, UNCERTAINTY MATRICES AND ROBERTSON INEQUALITY

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Abstract. The problem of diagonalization of Hamiltonians of N-dimensional boson systems by means of time-dependent canonical transformations (CT) is considered, the case of quadratic Hamiltonians being treated in greater detail. The unitary generator of time-dependent CT which can transform any Hamiltonian to that of a system of uncoupled stationary oscillators is constructed. The close relationship between methods of canonical transformations, time-dependent integrals of motion and dynamical symmetry is noted.

The diagonalization and symplectic properties of the uncertainty matrix for $2N$ canonical observables are studied. It is shown that the normalized uncertainty matrix is symplectic for the squeezed multimode Glauber coherent states and for the squeezed Fock states with equal photon numbers in each mode. The Robertson uncertainty relation for the dispersion matrix of canonical observables is shown to be minimized in squeezed coherent states only.

1. Introduction

The method of canonical transformations (CT) proved to be a fruitful approach in treating quantum systems. It is most efficient for systems that are described by Hamiltonians, that are quadratic in coordinates and momenta, or equivalently in boson creation and annihilation operators (quadratic Hamiltonians). The main advantage of the method of CT consists in reducing the Hamiltonian H of the treated system $\mathcal{S}$ to a Hamiltonian H' of some simple system $\mathcal{S}'$ with known solutions. The well known example (and probably the first one) of such an application is the diagonalization of the model quadratic Hamiltonians in superfluidity and superconductivity theory by means of linear time-independent