Twenty First International Conference on
Geometry, Integrability and Quantization
June 3–8, 2019, Varna, Bulgaria
Ivaïlo M. Mladenov, Vladimir Pulov
and Akira Yoshioka Editors
Avangard Prima, Sofia 2020, pp 57–65
doi: 10.7546/giq-21-2020-57-65

Geometry, Integrability and Quantization

NEW DEVELOPMENTS ON THE p-WILLMORE ENERGY OF SURFACES

EUGENIO AULISA, ANTHONY GRUBER, MAGDALENA TODA and HUNG TRAN

Department of Mathematics and Statistics, Texas Tech University
Lubbock, TX 79409, USA

Abstract. The p-Willmore energy $\mathcal{W}^p$, which extends the venerable Willmore energy by accommodating different powers of the mean curvature in its integrand, is a relevant geometric functional that bears both similarities and differences to its namesake. To elucidate this, some recent results in this area are surveyed. In particular, the first and second variations of $\mathcal{W}^p$ are given, and a flux formula is presented which reveals a connection between its critical points and the minimal surfaces. Finally, a model for the p-Willmore flow of graphs is presented, and this connection is visualized through computer implementation.

MSC: 53C44, 53C42, 53C21
Keywords: Curvature functionals, minimal surfaces, Willmore energy, Willmore flow

1. Introduction

An important conformal invariant of immersed surfaces in $\mathbb{R}^3$ is known as the Willmore energy

$$\mathcal{W}^2(\Sigma) = \int_\Sigma H^2 \, \mathrm{d}S$$

which measures the failure of the surface Σ to be totally umbilic. Besides being the object of much mathematical study in recent decades (e.g. [3, 8, 11, 17, 18] and references), the Willmore energy has also demonstrated its utility in fields such as biology and biophysics, where adaptations like the Helfrich model for biomembranes have proven to be highly accurate models of observable behavior [9].
Due to this mathematical and physical relevance, it is reasonable to consider an