



## ROTATIONS IN $\mathbb{R}^3$ AND THEIR PARAMETRIC REPRESENTATIONS

CLEMENTINA D. MLADENOVA AND IVAÏLO M. MLADENOV<sup>†</sup>

*Institute of Mechanics, Bulgarian Academy of Sciences, Acad. G. Bonchev Str. Block 4, 1113 Sofia, Bulgaria*

<sup>†</sup> *Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences, Acad. G. Bonchev Str. Block 21, 1113 Sofia, Bulgaria*

**Abstract.** The present paper is a review of the research in the area of representations of the rotational motions in the three-dimensional Euclidian space. The study starts with the topics of Euler angles and Rodrigues’ formula, and passes through the investigations in quaternion, spinor and vector kinematics. The authors present the interconnections between the different parameterizations of  $SO(3)$  group and stress on their merits and negative characteristics, and applications. A special attention is paid on the vector-parameterization of the rotation group, and how its nice properties are used in different mechanical applications.

### CONTENTS

1. Introduction . . . . .	187
2. Direction Cosines Defining a Rotation Matrix . . . . .	188
3. Euler Angles . . . . .	191
4. Rodrigues’ Formula . . . . .	193
4.1. Vector Form of the Rotation Tensor . . . . .	194
4.2. Coordinate Form of the Rotation Tensor . . . . .	194
4.3. Determination of $\mathbf{n}$ , $\vartheta$ and $\mathcal{R}$ . . . . .	195
5. Rigid-Body Kinematics . . . . .	196
5.1. Vector Kinematics . . . . .	197
5.2. Quaternion and Spinor Kinematics . . . . .	200
6. Vector Representation of Rotation Motions . . . . .	203