

## APPROXIMATE ANALYTICAL SOLUTIONS OF GENERALIZED LANE–EMDEN–FOWLER EQUATIONS

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**Abstract.** The work deals with a family of nonlinear differential equations of Lane–Emden–Fowler type. The original Lane–Emden equation was used to model the thermal behavior of spherical clouds of gas within the framework of classical thermodynamics. Slightly modified, it describes phase transitions in critical thermodynamic systems, cylindrical equilibrium shapes of lipid membranes and many other physical processes and phenomena. The aim of the present work is to obtain approximate analytical solutions of the regarded equations. The problem is formulated in terms of nonlinear Volterra integral equations of the second kind. The solutions are sought by He’s homotopy perturbation technique, as series expansion in the independent variable, and by Picard’s method of successive approximations.

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### 1. Introduction

The Lane–Emden equation [6, 10] is one of the basic equations in the theory of stellar structure. It has been the focus of many studies for almost 150 years now. This equation describes the temperature variation of a spherical gas cloud under the mutual attraction of its molecules within the framework of classical thermodynamics [1]. It also describes the variation of density as a function of the radial distance for a polytrope [13] and many other physical processes and phenomena in thermodynamics, continuum mechanics, astrophysics, etc.