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SOLITON SURFACE FOR THE (1+1)-DIMENSIONAL SCHRÖDINGER-MAXWELL-BLOCH EQUATION

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Abstract. In this paper, we study the (1+1)-dimensional Schrödinger-Maxwell-Bloch equation (NLS-MBE) which is integrable by the Inverse Scattering Method. Its Lax pair is presented. We apply methods of the theory of integrable systems to the geometry of surfaces immersed in Euclidean spaces. Using Sym-Tafel formula we construct the first and second fundamental forms, the Gaussian (total) curvature, mean curvature and Christoffel symbols for the NLS-MBE.
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Keywords: Fundamental forms, geometry of solitons, Lax pair, Schrödinger-Maxwell-Bloch equation, soliton surfaces, Sym-Tafel formula

1. Introduction

The interest in studying soliton surfaces of integrable models is motivated by various applications in the following areas: two-dimensional gravity [5], quantum field theory, string theory [7], in statistical physics: the Ising model [8], the theory of gauge fields, phase transition, (increase in the crystal lattice, deformation of membranes, surface waves, etc. [6,10,14]), in hydrodynamics: the movement of regions between different densities and viscosities [3].

Soliton equations play a significant role for the construction of surfaces. The theory of nonlinear soliton equations was developed in 1960s. Lax representation of integrable equations should exist in order to apply the Inverse Scattering Method for finding solutions of these integrable equations [1], [4]. Soliton surface theory was first developed by Sym [11]-[13].