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N^{th}-ORDER SUPERINTEGRABLE SYSTEMS SEPARATING IN POLAR COORDINATES

İSMET YURDUŞEN

Department of Mathematics, Hacettepe University, 06800 Beytepe, Ankara, Turkey

Abstract. Classical and quantum Hamiltonian systems in two-dimensional Euclidean plane and allowing separation of variables in polar coordinates are investigated. The additional integral of motion is assumed to be a polynomial of degree $N \geq 3$ in momenta. After analyzing the particular cases of $N = 3, 4$ and 5, a general description will be given. This leads to a classification of superintegrable potentials into two major categories. For the exotic potentials, the existence of an infinite family of superintegrable potentials in terms of the sixth Painlevé transcendent P_6 is conjectured and will be demonstrated for the first few cases.

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1. Introduction

Constructing and analyzing a mathematical model, using it to make predictions and comparing with an experiment usually constitute a standard way to analyze a physical system. Albeit the mathematical models constructed in this regard achieve a great success in classical and quantum mechanics, only relatively few of them can be solved exactly with explicit analytic expressions. These are called integrable Hamiltonian systems. A further special subclass is superintegrable systems, which admit the maximum possible symmetry and therefore can be solved algebraically as well as analytically. The proper definitions of them are given in [5, 17–19].

Definition 1. *In classical mechanics a Hamiltonian system with n degrees of freedom is called integrable, if it allows n functionally independent integrals of motion $\{X_1, \ldots, X_n\}$ in involution. Of course, Hamiltonian H belongs to this set.*