



NEW AND OLD PARAMETERIZATIONS OF THE CASSINIAN OVALS AND SOME APPLICATIONS OF THEM

IVAĀLO M. MLADENOV^{1,2}

Presented by IvaĀlo M. Mladenov

A plethora of new explicit formulas that parameterize all three types of the Cassinian ovals via elliptic and circular functions are derived from the first principles. These formulas allow a detailed study of the geometry of the Cassinian curves which is persuaded to some extent here. Conversion formulas relating various sets of the geometrical parameters are presented. On the way some interesting relationships satisfied by the Jacobian elliptic functions were found. Besides, a few general identities between the complete elliptic integrals of the first and second kind were also established. An explicit universal formula for the total area within the Cassinians which is valid for all types of them is derived. Detailed derivation of the formulas for the volumes of the bodies obtained as a result of rotations of the Cassinian ovals is presented.

MSC: 53A04, 51M25, 26B15

Keywords: Cassinian oval, identities between Jacobian elliptic functions, volumes of the rotated Cassinians, universal formula for the area of the Cassinians

Contents

1	Introduction	76
2	New Parameterizations	78
3	Geometry of the Cassinian Ovals	83
4	Areas within the Cassinians	85
5	Volumes	91
5.1	Volume of the Rotated Lemniscates	91
5.2	Volume of the Cassinian Tori	92
5.3	Volume of the Cassinian “Horns”	93
5.4	Volume of the Cassinian “Pears”	94
6	Comments	96
	References	96
	doi:10.7546/giq-23-2022-75-98	75