



LINEAGE OF THE THEORY OF INVARIANT INTEGRALS ON GROUPS

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From the standpoint of the History of Mathematics, beginning with pioneering work of Hurwitz on invariant integrals (or invariant measures) on Lie groups, we pick up epoch-making works successively and draw the main stream among so many contributions to the study of invariant integrals on groups, due to Hurwitz, Schur, Weyl, Haar, Neumann, Kakutani, Weil, and Kakutani-Kodaira, and explain their contents and give the relationships among them.

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Contents

1	Introduction	2
2	Hurwitz’s Pioneering Work on Integrals on Lie Groups	3
2.0	On ‘Introduction’	3
2.1	On ‘§ 1 Generation of Invariants on Continuous Groups Using Invariant Integral’	4
2.2	On ‘§ 2 Global Coordinates on $SO(n)$ and Formula of Invariant Measure’	6
2.3	Another Form of Global Coordinates on $SO(n)$	7
2.4	On ‘§ 3 Global Coordinates on $SU(n)$ and Formula of Invariant Measure’	8
2.5	On ‘§ 4 General Theory for Lie Groups’	10
3	Frobenius’ Foundation and Schur’s Simplification of Representation Theory	10
3.1	Frobenius’ First Three Papers	10
3.2	Schur’s Simplification of Character Theory for Finite Groups	15
4	Schur’s Work and Communications with Weyl	17
4.1	Schur’s Classification of Irreducible Representations of n -th Rotation Groups as Application of Invariant Integral	17
4.2	Weyl’s Idea Inspired by Schur (an Advanced Announcement)	20