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CLIFFORD ALGEBRAS, HYPERCOMPLEX NUMBERS AND NONLINEAR EQUATIONS IN PHYSICS

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Hypercomplex number systems are vector algebras with the definition of multiplication and division of vectors, satisfying the associativity and distributive law. In this paper, some new types of hypercomplex numbers and their fundamental properties are introduced, the Clifford algebra formalisms of hydrodynamics and gauge field equations are established, and some novel consistent conditions helpful to understand the properties of solutions to nonlinear physical equations are derived. The coordinate transformation and covariant derivatives of hypercomplex numbers are also discussed. The basis elements of the hypercomplex numbers have group-like properties and satisfy a structure equation $\mathbf{A}^2 = n\mathbf{A}$. The hypercomplex number system integrates the advantages of algebra, geometry and analysis, and provides a unified, standard and elegant language and tool for scientific theories and engineering technology, so it is easy to learn and use. The description of mathematical, physical and engineering problems by hypercomplex numbers is of neat formalism, symmetric structure and standard derivation, which is especially suitable for the efficient processing of the higher dimensional complicated systems.

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Contents

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