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STAR PRODUCT DEFORMATION OF GAMMA FUNCTION*

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One parameter deformation of gamma function is given by means of convergent star product, where the deformation parameter is a complex number with negative real part. Several basic identities of gamma function are deformed to give ones for the deformed gamma function.

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1. Introduction

Star product is given first in [1] as a deformation of usual multiplication of functions in the space of formal power series with respect to the deformation parameter, which gives an associative, noncommutative product. In [2], instead of the formal power series, the convergent star product is proposed and the function spaces are clarified where the star product is convergent. In [5], it was remarked that noncommutative and also commutative star products are given in the same framework.
Using the star product we can obtain a deformation of functions, called star functions in the following way. For expansion of an analytic function, we replace the usual multiplication of variables with the star product and obtain its deformed function if the deformed expansion is convergent. By means of the convergent star product in this way, we can discuss a problem of construction of star functions and also their functional relations (see for example [7, 8]).

In this short note, we deal with commutative star products and we discuss a commutative deformation of gamma function via star product, called the star gamma function. The idea is to use the star exponential function, the exponential function given by the usual expansion of exponential in terms of the star product. Using the star exponential function instead of the usual one, we define the star gamma function by means of the Euler's expression as an infinite integral. We discuss some properties of the star gamma function together with its functional identities.

*Dedicated to the memory of Fumio Ichikawa.