



# A PROOF OF THE SOLAR SYSTEM STABILITY FOR THE NEXT 100 000 YEARS

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Here we present an analytical proof assisted by computer calculations for the dynamical stability of the eight main planets and Pluto for the next 100 000 years. It means that the semi-major axes of the planets will not change significantly during this period. Also the eccentricities and the inclinations of the orbits will remain sufficiently small. A standard linear four-step numerical method is used to integrate approximately the orbits of Mercury, Venus, Earth+Moon, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Written in orbital elements, the dynamics of the nine planets manifests a system of 54 first-order ordinary differential equations. The step-size of the numerical method – about six days, has been performed 6 290 000 times. We estimate also the total accumulation of rounding-off errors, deviations related to possible uncertainty in the astronomical data and the accuracy of the computer calculations.

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