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INVESTIGATIONS ON SPECIAL SOLITONS FOR A GRADIENT EINSTEIN-TYPE SPACETIME

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In this paper, we study on a gradient Einstein-type spacetime. Under what special conditions such spacetimes will be transformed into solitons of special type are among the other topics of this study.
Considering a gradient Einstein-type spacetime with a parallel vector field, we establish the subsequent results.

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1. Introduction

A connected, n-dimensional ($n \geq 3$) Riemannian manifold (M, g) is called an Einstein-type manifold if there exists a smooth vector field $X \in \chi(M)$ and $\lambda \in C^\infty(M)$ such that

$$\alpha \mathtt{Ric} + \frac{\beta}{2}\mathcal{L}_X g + \mu X^b \otimes X^b = (\rho R + \lambda)g \tag{1}$$

for some constants α, β, μ, $\rho \in \mathcal{R}$ with the condition $(\alpha,\beta,\mu) \neq (0,0,0)$. Here, $\mathcal{L}$, X^b, $\mathtt{Ric}$ and R denote the Lie derivative, the one-form metrically dual to the vector field X, Ricci tensor and the scalar curvature of this manifold, respectively. If $X = \nabla f$ for some smooth functions $f\colon M \to \mathcal{R}$, we say that (M, g) is a gradient Einstein-type manifold [3]. Thus, equation (1) can be written as

$$\alpha \mathtt{Ric} + \beta \mathtt{Hess}(f) + \mu \mathrm{d}f \otimes \mathrm{d}f = (\rho R + \lambda)g \tag{2}$$

for some scalar functions $\alpha, \beta, \rho, \mu \in \mathcal{R}$ and Hess stands for the Hessian.

A complete Riemannian manifold (M, g) is said to be a Ricci soliton if the condition

$$\mathtt{Ric} + \frac{1}{2}\mathcal{L}_X g = \lambda g \tag{3}$$