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ON INFORMATION GEOMETRY METHODS FOR DATA ANALYSIS

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The paper is devoted to exploration of statistical manifolds. In particular we consider exponential family manifolds, its metrics and connections. We proved that a statistical manifold does not admit nontrivial geodesic mapping between the Riemannian connection and any alpha-connection. Besides, a statistical manifold does not admit a Weyl connection which coincides with any alpha-connection.
We obtain also that the Fisher information matrix calculated for a mixture family is a Hessian metric.

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1. Introduction

Information geometry as a subject emerged in the fundamental paper [9]. There it has been suggested to consider the Fisher information matrix as a metric of some smooth manifold M. The manifold is constructed from a parameterized family of probability distributions and the Fisher information matrix plays the role of the manifold Riemannian metric. Rao and Cramér [5] presented independently of each other a fundamental theorem in statistics, which was later called the Cramér-Rao theorem. So the Riemannian distance between two Gaussian distributions with different means and variances was calculated using the Fisher metric.

Rao [10] proposed also a higher-order asymptotic theory of statistical inference, according which the maximum-likelihood estimator (MLE) keeps the maximum amount of information up to higher orders. Also he introduced a third-order symmetric tensor. Both the Fisher metric and third-order symmetric tensor define invariant affine connections on a statistical manifold M.

One knows that expanding the estimation error in series of $O(1/n)$ presents the MLE as an optimal estimator if number of observations n is large. Obviously the ‘higher order’ implies terms of $O(1/n^2)$. A geometrical theory which describes the higher-order quantities through statistical curvatures was proposed by Efron [6].

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