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DUALLY FLAT STRUCTURE FOR PARAMETRIC FAMILIES OF GROUND STATES OF QUANTUM SYSTEMS

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The dually flat structure is a central notion in information geometry. In this paper, we introduce a Bregman divergence and hence a dually flat structure to a ground-state manifold, the parametric family of the ground states of a parameterized quantum Hamiltonian. Taking an Ising chain as an example, we also discuss an approximation of a quantum Hamiltonian by the projection theorem.

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1. Introduction

Information geometry [2, 3, 9] provides statistical models with geometric perspectives and hence is applied to various fields of physics such as phase transitions and critical phenomena in statistical physics with a focus on the scalar curvature of the Fisher metric and the Levi-Civita connection [4, 7, 10]. One of the most important notions in information geometry is, apart from the Levi-Civita connection, the dualistic structure by dual connections and particularly, its flat case, which we call the dually flat structure. The dually flat structure has a wide range of applications from approximations or estimations of model parameters [12] to gradient systems [14].

One useful method to introduce a dualistic structure to a manifold is by a divergence [6]. In particular, a Bregman divergence, the divergence induced by a strictly convex function, can induce a dually flat structure. In other words, if we are naturally given a strictly convex function on a manifold, we can induce a dually flat structure [1]. The exponential family is one such example.

Let us focus our attention on the quantum case, where geometric methods are also studied and applied, for example, for investigating entanglement from a Riemannian geometric perspective [13]. In particular, the dually flat structure of the (full) state space in a finite dimension has been discussed and is found to be closely related to the so-called Bogoliubov-Kubo-Mori metric and the quantum relative