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GRADIENT EINSTEIN-TYPE SPACETIMES

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In the first part of this article, gradient Einstein-type manifolds are defined. In the second part, properties of a perfect fluid spacetime are given. In the third part, considering gradient Einstein-type spacetimes under the special conditions, some theorems are proved.

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1. Introduction

Assume that (M, g) is a connected Riemannian manifold of dimension n $(n > 3)$. We denote $\mathtt{Ric}$ and R as its corresponding Ricci tensor and scalar curvature, respectively. (M, g) is called an Einstein-type manifold (it means that (M, g) supports an Einstein-type structure) if there exist a smooth vector field X on M and a smooth function $\lambda\colon M \to \mathcal{R}$ satisfying the condition

$$\alpha\mathtt{Ric} + \frac{\beta}{2}\mathcal{L}_X g + \mu X^b \otimes X^b = (\rho R + \lambda)g \tag{1}$$

for some constants $\alpha, \beta, \mu, \rho \in \mathcal{R}$ with the condition $(\alpha, \beta, \mu) \neq (0, 0, 0)$. Here, $\mathcal{L}$, X^b, $\mathtt{Ric}$ and R denote the Lie derivative, the one-form metrically dual to the vector field X, Ricci tensor and the scalar curvature of this manifold, respectively.

If $X = \nabla f$ for some smooth functions $f\colon M \to \mathcal{R}$, we say that (M, g) is a gradient Einstein-type manifold [2]. Thus, equation (1) can be written as

$$\alpha\mathtt{Ric} + \beta\nabla^2 f + \mu \mathrm{d}f \otimes \mathrm{d}f = (\rho S + \lambda)g \tag{2}$$

where $\nabla^2 f$ stands for the Hessian of f. Here, we refer to f as the potential function.

Let (M, g) be an n-dimensional semi-Riemannian manifold whose metric g is of signature (r, s) such that $r + s = n$. Choosing signatures of type $(r, s) = (1, n - 1)$ or $(r, s) = (n - 1, 1)$, M is said to be a Lorentzian manifold [9]. A time-oriented,

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