ON THE MULTIPLICITY OF CERTAIN YAMABE METRICS

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Abstract. In this paper we are interested in the multiplicity of the Yamabe metrics for compact Riemannain manifolds. This problem may be approached only when the scalar curvature is positive. This is the case notably of certain locally conformally flat manifolds. Furthermore, there exists a link with the singular Yamabe problem. We give various applications of this study.

1. Introduction

Consider a smooth compact Riemannian manifold (M, g) of dimension $n \geq 3$ where g is a smooth Riemannian metric on M, $\mathrm{d}v$ denotes the volume form of g, and $R(x)$ its scalar curvature. $\mathcal{H}_1^2(M, g)$ is the Sobolev space. For any nonzero function $u \in \mathcal{H}_1^2(M, g)$, we define the functional

$$J(u) = \frac{4\dfrac{n-1}{n-2} \displaystyle\int_M |\nabla u|^2 \, \mathrm{d}v + \displaystyle\int_M R(x)u^2 \, \mathrm{d}v}{\left(\displaystyle\int_M u^{\frac{2n}{n-2}} \, \mathrm{d}v\right)^{\frac{n-2}{n}}} \, . \tag{1}$$

The critical points of the functional $J(u)$ are solutions of the Euler–Lagrange equation, called Yamabe equation

$$4\frac{n-1}{n-2}\Delta_g u + R(x)u = K|u|^{\frac{4}{n-2}} u \, , \tag{2}$$

where K is a real number.