Sixth International Conference on
Geometry, Integrability and Quantization
June 3–10, 2004, Varna, Bulgaria
Ivaïlo M. Mladenov and Allen C. Hirshfeld, Editors
SOFTEX, Sofia 2005, pp 203–217

ON THE MULTI-COMPONENT NLS TYPE EQUATIONS ON SYMMETRIC SPACES: REDUCTIONS AND SOLITON SOLUTIONS

GEORGI G. GRAHOVSKI[†], VLADIMIR S. GERDJIKOV[†]
and NIKOLAY A. KOSTOV[‡]

[†]*Institute for Nuclear Research and Nuclear Energy*
Bulgarian Academy of Sciences
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria
[‡]*Institute of Electronics, Bulgarian Academy of Sciences*
72 Tsarigradsko chaussee, 1784 Sofia, Bulgaria

Abstract. The fundamental properties of the multi-component nonlinear Schrödinger (MNLS) type models related to symmetric spaces are analyzed. New types of reductions of these systems are constructed. The Lax operators L and the corresponding recursion operators Λ are used to formulate some of the fundamental properties of the MNLS-type equations. The results are illustrated by specific examples of MNLS-type systems related to the **D.III** symmetric space for the $\mathfrak{so}(8)$-algebra. The effect of the reductions on their soliton solutions is outlined.

1. Introduction

The (scalar) **nonlinear Schrödinger** (NLS) **equation** [26]

$$\mathrm{i}u_t + u_{xx} + 2|u|^2 u = 0, \quad u = u(x,t) \tag{1}$$

has numerous applications in a wide variety of physical problems [8, 24]. Its complete integrability has been proven at the very early stage of the inverse scattering method (ISM). Indeed, equation (1) allows Lax representation using as Lax operator the 2×2 **Zakharov–Shabat system** related to $\mathfrak{sl}(2)$-algebra

$$L(\lambda)\psi(x,t,\lambda) = (\mathrm{i}\partial_x + q(x,t) - \lambda\sigma_3)\psi(x,t,\lambda) = 0 \tag{2}$$
$$q(x,t) = \begin{pmatrix} 0 & u(x,t) \\ -u^*(x,t) & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$