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# DIMENSION FORMULAS FOR AUTOMORPHIC FORMS OF COABELIAN HYPERBOLIC TYPE

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**Abstract.** There are infinitely many hyperbolic transforms of complex abelian surfaces. The corresponding universal covers change from the complex plane to the unit ball, from flat to hyperbolic metrics. Looking back to Jacobi's periodic functions we were able to construct 2-dimensional abelian functions transformable to automorphic forms on the ball. In this article we prove explicit dimension formulas for all forms of this coabelian types for fixed weights.

## 1. Motivations and Main Results

The construction of hyperbolic surfaces by blowing up points of special abelian surfaces and contracting some elliptic curves has been published recently in [10]. For explicitly known cases the corresponding fundamental groups on the ball are of arithmetic nature. They are called Picard modular groups. For Jacobi-type construction and transfer of abelian functions to automorphic forms we refer to [9]. In the explicitly known cases they are called Picard modular forms. Comparing algebraic structures in [9] it seems to be that we found almost all Picard modular forms of this type. To clarify the situation it is necessary to know the dimensions of all these Picard modular form spaces of any given weight. This is the purpose of this paper.

The question of finding all of then remains to be an open problem. Moreover, the complete algebraic ring structure should be clarified. To be more precise, we look for explicit structures of rings $R(\Gamma)$ of modular forms for Picard modular groups $\Gamma$, especially in cases when the corresponding Picard modular surfaces are well determined by explicitly known algebraic equations. The quotient surface $\Gamma\diagdown\mathbb{B}$, $\mathbb{B}$ the complex two-dimensional unit ball, can be compactified by means of finitely

In memory to C. G. Jacobi (10 December 1804).