

HOW MANY TYPES OF SOLITON SOLUTIONS DO WE KNOW?

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Abstract. We discuss several ways of how one could classify the various types of soliton solutions related to NLEE that are solvable with the generalized $n \times n$ Zakharov–Shabat system. In doing so we make use of the fundamental analytic solutions, the dressing procedure and other tools characteristic for the inverse scattering method. We propose to relate to each subalgebra $\mathfrak{sl}(p)$, $2 \leq p \leq n$ of $\mathfrak{sl}(n)$, a type of one-soliton solutions which have $p - 1$ internal degrees of freedom.

1. Introduction

It is our impression that the question in the title has not been answered satisfactorily even for some of the best known type of soliton equations such as the N -wave equations, the multicomponent **nonlinear Schrödinger (NLS) equation** and others.

We are using the term “soliton solution” as a special solution to a given **nonlinear evolution equation (NLEE)** which is solvable by the so called **inverse scattering method (ISM)** [20, 4]. That means that the NLEE allows **Lax representation**

$$[L(\lambda), M(\lambda)] = 0 \quad (1)$$

where $L(\lambda)$ and $M(\lambda)$ are two linear operators. In what follows we take them to be first order matrix differential operators

$$L\psi(x, t, \lambda) \equiv i \frac{d\psi}{dx} + U(x, t, \lambda)\psi(x, t, \lambda) = 0 \quad (2)$$