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MANEV PROBLEM AND ITS REAL FORM DYNAMICS: SUPERINTEGRABILITY AND SYMMETRY ALGEBRAS

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Abstract. The Manev model is known to possess Ermanno–Bernoulli type invariants similar to the Laplace–Runge–Lenz vector of the ordinary Kepler model. If the orbits are bounded these invariants exist only when a certain rationality condition is met and consequently we have superintegrability only on a subset of initial values. On the contrary, real form dynamics of the Manev model is superintegrable for all initial values. Using these additional invariants, we demonstrate here that both Manev model and its real Hamiltonian form have $\mathfrak{su}(2) \simeq \mathfrak{so}(3)$ (or $\mathfrak{so}(2,1)$ depending on the value of a parameter in the potential) symmetry algebra in addition to the angular momentum algebra. Thus Kepler and Manev models are shown to have identical symmetry algebras.

1. Introduction

Since Kepler and Newton elliptical trajectories replaced circular ones as an archetype of the (bounded) planetary motion. The advent of Einstein's theory did not produce a new archetype of heavenly motions, apart from the exceptional case of a collapse into the (still hypothetical) black holes. Nevertheless, among the variety of relativistic effects the perihelion shift of inner planets is definitely the best recognizable effect in the Solar system. Maybe it is a time to accept a new archetype of heavenly motions – *precessing ellipse* (or more generally, precessing