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EXTERIOR DIFFERENTIAL SYSTEMS AND BILLIARDS

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Abstract. I describe work in progress with Baryshnikov and Zharnitsky on periodic billiard orbits that leads one to an exterior differential system (EDS). I then give a brief introduction to EDS illustrated by several examples.

1. Introduction

The purpose of these notes is to introduce the reader to the techniques of **exterior differential systems** (EDS) in the context of a problem in billiards. The approach in this article is different from that of [13] and [16], which begin with a study of linear Pfaffian systems, an important special class of EDS. The billiard problem results in an EDS that is not a linear Pfaffian system, so these notes deal immediately with the general EDS. For the interested reader, two references regarding EDS are [6] and [13]. The first is a definitive reference and the second contains an introduction to the subject via differential geometry. For more details about anything regarding EDS the reader can consult either of these two sources. Cartan's book on EDS [10] is still worth looking at, especially the second half, which is a series of beautiful examples.

We generally will work in the real analytic category, although all the non-existence results discussed here imply non-existence of smooth solutions.

Notation

If M is a differentiable manifold we let TM, T^*M denote its tangent and cotangent bundles, $\Omega^d(M)$ the set of differential forms on M of degree d and $\Omega^*(M) = \oplus_d \Omega^d(M)$. If $I \subset T^*M$ is a subbundle (more precisely, subsheaf), then we let $\{I\}_{\rm diff} \subset \Omega^*(M)$ denote the differential ideal generated by I, i.e, all elements of $\Omega^*(M)$ of the form $\alpha \wedge \phi + {\rm d}\beta \wedge \psi$ where $\alpha, \beta \in I$ and $\phi, \psi \in \Omega^*(M)$.